

Unified Field Theory

Mendel Sachs

The subject, “unified field theory”, has been frequently referred to in the scientific literature over the past centuries. Yet it is not generally understood what this means and why it is significant for science. My purpose in this essay is to explain what a unified field theory refers to, for the lay readers of science as well as the professional physicists.

We must first ask: What is a “field theory”? The basic idea was introduced by Michael Faraday in the 19th century. It was a paradigm change in regard to the view we have of matter. It is a view that is opposite to that of ‘atomism’, that pervaded the thinking of physicists and philosophers since the times of ancient Greece. The field view of matter is a continuous, holistic approach, in fundamental terms, rather than considering a material system as a collection of singular, ‘things’, interacting with each other in space. The things that we encounter – protons, people, planets, stars, galaxies, ... – are an infinite distribution of the (correlated) modes of this continuum, that in general is the universe.

In the 19th century, the predominant thinking about electricity was that there are individual sources of electrical force - positively or negatively charged bodies – that affect other charged bodies a distance away. The strength of this electrical force between charged bodies depends on the inverse square of the distance between them and on their intrinsic charges e and e' . If e and e' have opposite signs the force is attractive; if they have the same sign, the force is repulsive. The idea in this atomistic view is that there is a singular source of electrical force that depends on its intrinsic charge, e . The

divergent set of lines of force that emanate from this particle of matter are continuously distributed in space. If the second body, with the charge e' is on a line of force, r cm from e , then the force between them depends on ee'/r^2 .

In this atomistic view, then, the electrical force is due to a singular electrically charged body with an accompanying set of lines of potential force, to act on another body that is r cm from e . If e has a positive value, a negatively charged body $-e'$ will move along the line of force toward e . A positively charged body $+e'$ will move away from e along this same line of force.

It is interesting to note, topologically, that since a line has two ends, the existence of a positive charge, at one end of the line, implies the existence of a negative charge at the other end. That is, a 'test body' with the same sign of charge as the 'beginning' of this line, will move toward the other end of the line, with opposite charge. Thus the prediction follows that for every positively charged body in the world, a negatively charged body must exist.

The topology of the magnetic lines of force is different than that of the electrical source. Here we have opposite poles of a magnet, always in proximity. If one magnet is placed near another magnet, its south pole, s , will align itself toward the north pole N of the first magnet. Its north pole, n , will align itself toward the south pole S of the first magnet. Since N and S are in proximity, the magnetic lines of force of the first magnet must be curved, starting at N and ending at S . This is in contrast with the straight divergent lines of potential force of an electrical source.

The ‘field concept’ envisioned by Faraday is that it is the continuum of (magnetic or electrical) lines of force, *a field*, that is the fundamental representation of matter, rather than the particles of matter that were previously believed to generate this field. The sources, electrical charged body or a magnetic body, are then manifestations of the fields, rather than the other way around.

While the electrical and magnetic fields of force have different configurations, Faraday felt that they must be aspects of a *unified electromagnetic field*. This idea had empirical backing from the experiment of H. C. Oersted. What he saw was that an electrical current in a straight wire induced a magnetic field of force in a plane that is perpendicular to the direction of the current carrying wire. This is seen in the polarization of small magnets (such as compass needles) in the perpendicular plane in a circular pattern surrounding the wire. Further, a reversal of the direction of the current in the wire caused the magnets in the plane perpendicular to the wire to reverse their orientation by 180 degrees. Thus, Oersted’s experiment showed that a magnetic field is an electrical field *in motion* (an electric current). But since motion, per se, is subjective, it follows that an electrical field is a magnetic field *in motion*. Thus Faraday, with Oersted, concluded that the objective field that underlies electric and magnetic phenomena is a *unified electromagnetic field*. It then follows that if one should view a static electric field, no magnetic field would be observed under these conditions. But if the source of the electric field should be in motion, a magnetic field would be detected as well. The idea of the unified field theory was then proposed for all forces in nature, generalizing from the case of electromagnetism.

Faraday tried to show, empirically, that the gravitational field of force is included in the unified electromagnetic field, but he was not able to demonstrate this. This was primarily because of the large magnitude of the electromagnetic coupling between matter compared with the gravitational coupling, in the domain that Faraday studied. In the theory of general relativity that Einstein proposed, there are logical reasons to believe that all forces in nature - the gravitational, electromagnetic, nuclear and weak forces - must be unified in a single field of force.

Einstein's Theory of General Relativity and Holism

The most important physics program in the 20th century that continued Faraday's quest for a unified field theory was Einstein's theory of general relativity. To understand why this theory leads to the requirement of a unified field theory we will first discuss the paradigm shift from the atomistic view to the continuum, holistic field approach.

First, however, to define Einstein's theory it should be pointed out that his notion of a field theory extends from Faraday's. Faraday's field theory is still based on an open system – fields and the test particles that probe it. But in Einstein's field theory we have a closed system from the outset. In principle there are no discrete particles, even test bodies that are things in themselves. In asymptotic limits, portions of the total field approach the description of 'test bodies' that probe the field. Indeed it is this limit that allows the scientist to study the closed system. But it is important that the totally closed system is both conceptually and mathematically distinct from the view in which there are free test bodies at the outset. The view of a closed system

described by holism and continuous fields was Einstein's meaning of a field theory.

It is interesting that in a letter to David Bohm, in 1952, three years before Einstein passed away, he said the following: "When one is not starting from the correct elementary concepts, if for example, it is not correct that reality can be described as a continuous field, then all my efforts are futile"[Einstein Archives, Jewish National and University Library, The Hebrew University of Jerusalem, Call No. 4 1576:8-053]

The theory of general relativity is based on a single axiom: *the principle of relativity*. This is an assertion of the *objectivity* of the laws of nature. That is to say, from the perspective of any observer (a frame of reference), the laws of nature as expressed in all other frames of reference relative to that of his own must be in *one-to-one correspondence*. The usual (but not the only!) language that is used to express the laws of nature entails four continuous parameters – three are associated with a measure in three-dimensional space and the fourth is associated with the time measure. It is found that the objectivity of the laws of nature with this language implies that a spatial measure in one reference frame (perspective) must entail a particular mixture of spatial and temporal measures in any other reference frame where the laws are expressed. Thus a (subjective) measure of space and/or time becomes an (objective) measure of spacetime.

The spacetime language for the laws of nature entails continuous and continuously differentiable parameters. In mathematical language, these continuous parameters are the *independent variables*. The solutions of the laws of nature are *mapped* in this space of independent variables. These solutions,

in turn, are the continuous field variables that solve the laws of nature – the field equations. These are the *dependent variables*.

Thus we see that the solutions of the laws of nature are a set of continuous field variables. It follows further that these field variables must also be analytic. This follows from the requirement of the inclusion of the laws of conservation of energy, momentum and angular momentum, (in the special relativity limit) according to Noether’s theorem. Thus, it is a requirement of this field theory that the transformations of fields from one reference frame to another, to preserve the forms of the laws, are continuous and analytic. (*This implies that the spacetime coordinates in one reference frame are differentiable to all orders with respect to the spacetime coordinates of all other reference frames*). Such a transformation group is a “Lie group”. The mathematical functions subject to these requirements are called “regular”. The Lie group of the theory of special relativity theory (expressed in a flat spacetime) is the “Poincare group”. The Lie group of the theory of general relativity theory (expressed in a curved spacetime) is the “Einstein group”.

We see, then, that the theory of relativity predicts that the matter of the universe, in any of its domains, must be represented by a continuum, *holistically*. The apparent “things” that we respond to - protons, people, planets, stars, galaxies.. - are of the infinite distinguishable (correlated) *modes* of this continuum that in principle is the universe. The way in which these *modes* interact with each other in terms of the types of force, must depend on the physical circumstances of their coupling in a continuous way. That is, there are no sharp cut-offs where one type of force or another turns off and another turns on, discretely. Thus it is a feature of the continuous, holistic

field approach in general relativity that all forces exerted by matter on matter, *as well as the reactions to these forces*, must be the components of a unified field – just as Faraday had anticipated in the 19th century. The reaction of matter to the forces exerted on it by *all other matter* of a closed system must entail the inertial mass of matter in the full expression of the unified field theory. This is in accord with the *Mach principle*.

Summing up, the theory of general relativity, as a fundamental explanation of the true nature of matter, is a holistic, deterministic, continuous unified field theory. Its holism denies the particularity of matter, as it was assumed from ancient Greece through the period of elementary particles in present day physics.

There are present day claims in elementary particle physics that search for a “unified theory” – such as the “grand unified theory”. But these are not unified in the sense that we have discussed in this essay. They are attempts to generalize the categories of data with more “boxes”, as it is done in biology. This is a phenomenological approach that does not have the meaning of unification intended by Faraday and Einstein as we have discussed above.