The Mach Principle and the Origin of Inertia From General Relativity

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There has been a great deal of discussion during the 20^{th} century on the possible entailment of the Mach principle in general relativity theory. Is it a necessary ingredient? Additionally, there has been the question of the origin of the inertia of matter in general relativity – does inertia originate from the foundation of general relativity theory as an underlying theory of matter? I wish to demonstrate in this lecture that indeed both of these features of matter are intimately related to the conceptual and mathematical structures of the theory of general relativity.

The Theory of General Relativity

The first thing that we must do, then, is to clearly define terms. What do we mean by "the theory of general relativity"? I should like to preface this discussion with the comment that the title of the theory of relativity should be: "the theory of general relativity" (or the "theory of special relativity") rather than the more commonly used title: "the general theory of relativity", (or the "special theory of relativity") since it is the 'relativity' that is general (or special) and not the theory! There is indeed one theory of relativity, whether it is in the 'special' or the 'general' form, based on the single 'principle of covariance' (also called the 'principle of relativity'). The adjectives 'special' or 'general' refer to the types of relative motion of the frames of reference in which the laws are to be compared from the perspective of any one of them. When the relative motion is inertial, we have special relativity and when it is generally nonuniform we have general relativity. Thus it is the ‘relativity’ that is special or general, not the theory - which is a single concept!

The ‘principle of covariance’ is the underlying axiom that defines this theory. It is an assertion of the objectivity of the laws of nature, asserting that their expressions are independent of transformations to any frame of reference in which they are represented, with respect to any arbitrary observer’s perspective (frame of reference). This implies an entailment of all possible frames of reference; thus it implies that any real system of matter is a *closed system*. Of course, when the coupling between any local component of the closed system is sufficiently weakly coupled to the rest of the system, say to the rest of the universe or to any smaller subsystem of matter, then one may use the mathematical approximation in which all that there is to represent, mathematically, is the localized material system. In the next approximation, the rest of the system could perturb it. But for the actual *unapproximated* closed system, the implication is that there is no singular, separable ‘thing’ of matter. Any constituent matter is always relative to other components that together with it makes up the entire closed system – not as singular ‘parts’, but rather as the modes of a single continuum.

In fundamental terms, then, the principle of covariance implies, ontologically, a *holistic model*, wherein there are no individual, singular, separable things; the closed system is rather a single system without independent parts! This is also an implication of the definition of the inertial mass of matter, according to the *Mach principle*, which we will discuss in detail later on.

The model of matter we have come to, then, from the principle of covariance of the theory of general relativity, is one of *holism*. What we observe as individual separable ‘things’, that we call ‘elementary particles’ or ‘atoms’ or ‘people’ or ‘galaxies’, are really each correlated modes of a single continuum. The peaks of these modes are seen to move about and to interact with each other. But indeed they are not independent, separable things, as they are all correlated through the single matter continuum, of which they are its manifestations. This single continuum is, in principle, the universe.

The Mach Principle

We have seen that the qualities of localized matter, such as the inertial mass or electric charge of 'elementary particles', are really only measures of their interactions within a closed system of matter, between these entities and the rest of the system. Thus their values are dependent, numerically, on the rest of the matter of the closed system, of which they are elementary, inseparable constituents. Their masses and electric charges are then measures of coupling within a closed system, not intrinsic properties of 'things' of matter. The dependence of the inertial mass of localized matter, in particular, on the rest of the matter of the 'universe', is a statement of the Mach principle.

It should be emphasized, however, that what Mach said about this was not the commonly stated definition of the principle. The latter is the assertion that only the distant stars of the universe determine the mass of any local matter. In contrast to this, in his *Science of Mechanics*[1] Mach said that *all* of the matter of the universe, *not only the distant stars*, determines the inertial mass of any localized matter.

I have found in my research program in general relativity, that the primary contribution to the inertial mass of any local elementary matter, such as an 'electron', are the nearby particle-antiparticle pairs that constitute what we call the 'physical vacuum'. [The main developments of this research are demonstrated in my two monographs: *General Relativity and Matter*[2], and *Quantum Mechanics from General Relativity*[3]]. A prediction of this research program is that the main influence of these pairs on the mass of, say, an electron comes from a domain of the 'physical vacuum' in its vicinity, whose volume has a radius that is the order of 10^{-15} cm. Of course, the distant stars, billions of light-years away, also contribute to the electron's mass, though negligibly, just as the Sun's mass contribution to the weight of a person on Earth is negligible compared with the Earth's influence on this person's weight! Nevertheless, it was Mach's contention that in principle *all* of the matter of the closed system – the nearby as well as far away constituents – determines the inertial mass of any local matter.

Newton's Third Law of Motion

I believe that the first indication in physics of the holistic view of a closed material system came with *Newton's third law of motion*. I see this law as a very important precursor for the holistic aspect of Einstein's theory of relativity. The assertion of this law is that for every action (force) exerted on a body *A*, by a body *B* (that is located somewhere else), there is an equal quantity of (reactive) force exerted by *A*, oppositely directed, on *B*. According to this law of motion, then, the minimal material system must be the two-body system *A-B*. *A or B*, as individual, independent 'things', then loses meaning since, with this view, the limit in which *A* (or *B*) is by itself an entity in the universe does not exist!

One other mathematical feature (that was not noted by Newton) that is implied by his third law of motion is that the laws of motion of matter must be fundamentally *nonlinear*. For if *A*'s motion is caused by a force exerted on it by *B*, which in turn depends on *B*'s location relative to *A*, then the reactive force exerted by *A* on *B*, according to Newton's third law of motion, causes *B* to change its location relative to *A*. Consequently *B*'s force on *A* changes. Thus *A*'s motion would be changed from what it was without the reactive force on *B*. We must conclude, then, that *A*'s motion affects itself by virtue of the intermediate role that is played by *B* in the closed system *A-B*. The mathematical implication of this effect is in terms of *nonlinear laws of motion* for *A* (as well as for *B*). Thus we see that, at the foundational level, the model of matter, even in Newton's classical physics of 'things', must be in terms of a closed system that obeys nonlinear mathematical laws of motion.

The Generalized Mach Principle

As we will see later on, the principle of covariance of the theory of general relativity implies that the basic variables of the laws of matter must be continuous, nonsingular fields, *everywhere*. The laws of matter must then be a set of coupled, nonlinear field equations for all of the manifestations of the closed continuum that in principle is the 'universe'. Thus we see here how the Mach principle is entirely intertwined with the theory of general relativity, regarding the logical dependence of the inertial mass of local matter on a closed system.

The theory of general relativity goes beyond the Mach principle. It implies that *all* of the qualities of local matter, not only its inertial mass, are measures of dynamical coupling between this 'local' matter and the rest of the closed material system, of which it is a constituent. I have called this "the generalized Mach principle"[2]. Thus the foundational aspects of the theory of general relativity imply an ontological view of holism wherein all remnants of 'atomicity' are exorcised. With this view, the 'particle' of matter, as a discrete entity, is a fiction. What these 'things' are, in reality, are manifestations (modes) of a single matter continuum.

Let us now discuss the role of space and time in general relativity theory. We will then go on to show how the inertial mass of elementary matter emerges from the field theory of general relativity. Finally, it will be seen how the formal expression of quantum mechanics (the Hilbert space formalism) emerges as a linear approximation for a nonlinear, generally covariant field theory of the inertia of matter.

The Role of Space and Time in Relativity Theory

The assertion of the *principle of covariance* entails two scientific (i.e. in principle refutable) assertions. One is the existence of laws of all of nature. This is the claim that for every effect in nature there is a logically connected cause. This assertion is sometimes referred to as the 'principle of total causation'. These relations between causes and effects are the laws of nature that the scientists seek.

The second implication of the *principle of covariance* is that the laws of nature can be comprehended and expressed by us. This is, of course, not a necessary truth. But *as scientists*, we have faith in its veracity. The *expressions* of the laws of nature are where space and time come in. In this view, space and time are not entities in themselves. Rather they provide the 'words' and the logic of a language that is invented for the sole purpose of facilitating an expression of the laws of nature. *It is important to know that the concepts entailed in the laws of nature underlie their language expressions – in one expression or another.*

The space and time parameters and their logic then form an underlying grid in which one *maps* the field solutions of the mathematical expressions of the laws of nature. The logic of the languages of the laws of nature is in terms of geometric and algebraic relations, as well as topological relations in some applications. With the assumption that a space and time grid forms a continuous set of parameters, the solutions of the laws of nature are then continuous functions of these parameters. These are the 'field variables'. They are the solutions of the 'field equations', field relations that are continuously *mapped* in space and time. According to the *principle of covariance*, the field equations must maintain their forms when transformed to *continuously connected* spacetime frames of reference.

It might be mentioned here, parenthetically, that there is no logical reason to exclude a starting assumption that the language of spacetime parameters is a discrete, rather than a continuous grid of points. In this case the laws of nature would be in the form of difference equations rather than differential equations. However, the implications of the spacetime parameters as forming a continuum, in the expressions of the laws of nature, as continuous field equations, agrees with all of the empirical facts about matter that we are presently aware of. Thus, we assume at the outset that the spacetime language is indeed in the form of a continuous set of parameters. Its geometrical logic in special relativity is Euclidean and in general relativity it is Riemannian. The algebraic logic is in terms of the defining symmetry group of the theory of relativity; it is a *Lie group* – a set of continuous, analytic transformations. The reason for the requirement of analyticity of the transformation group will be discussed in the next paragraph. The Lie group in special relativity is the 10-parameter *Poincare group*; in general relativity it is the 16-parameter *Einstein group*.

A requirement of the spacetime language, stressed by Einstein, as mentioned above, is that the field solutions of the laws of nature – the solutions of the 'field equations' – should be *regular*. This is to say, they should not only be continuous but also analytic (continuously differentiable to all orders, without any singularities) *everywhere*. I am not aware that Einstein gave any explicit reason for this requirement in his writings. However, I believe that it can be based on the *empirical requirement* that the (local) flat spacetime limit of the general field theory in a curved spacetime, must include laws of conservation – of energy, linear momentum and angular momentum. For, according to *Noether's theorem*,[4] the analyticity of the field solutions is a necessary and a sufficient condition for the existence of these conservation laws. Strictly, there are no conservation laws in general relativity because, covariantly, a 'time rate of change' of some function of the spacetime coordinates in a curved spacetime cannot be separated from the rest of the formulation that can go to zero. Thus, the *laws of conservation* apply strictly only to the local domain. The conservation laws are then a local limit of global laws in general relativity. In the latter global field laws, a time rate of change

can no longer be separated, by itself, from a four-dimensional differential change of functions mapped in a curved spacetime. That is to say, in the curved spacetime the continuous transformations of a purely time rate of change of a function of the space and time coordinates, from its frame of reference where it may appear by itself, to any other continuously connected frame of reference, leads to a mixture of space and time differential changes. In this case we cannot refer to an *objective* conservation (in time alone) of any quantity, in the curved spacetime.

Thus we see that, based on the foundations of the theory of general relativity, we have a closed, nonsingular, holistic system of matter. It is characterized by the continuous field concept wherein the laws of nature are expressed in terms of nonlinear field equations that maintain their forms under transformations between any continuously connected reference frames of spacetime (or other suitably chosen) coordinates. Their field solutions – the ‘dependent variables’ - are *regular functions* of the space and time parameters, that is to say they are continuous and analytic (nonsingular) *everywhere*. The space and time parameters and their logical relations form the language of the ‘independent variables’ in which the field variables are mapped. The generalized Mach principle is then a built-in (derived) feature of this holistic field theory in general relativity.

Inertia and Quantum Mechanics from General Relativity

Thus far I have argued that the (generalized) Mach principle is automatically incorporated in the (necessarily) holistic expression of the theory of general relativity, as a general theory of matter. I now wish to show how, in particular, the inertial mass of matter enters this theory of matter in a fundamental way. I will try to avoid, as much as possible, the mathematical details of this derivation. They are spelled out in full in my two monographs[2,3].

In my view, the revolutionary and seminal experimental discovery about matter that relates to the basic nature of its inertia was made 75 years ago, when it was seen that, under particular conditions, particles of matter, such as electrons, have a wave nature. These were the experimental discoveries of electron diffraction by Davisson and Germer, in the US, and independently by G.P. Thomson, in the UK.[5] What they observed was that electrons scatter from a crystal lattice with a diffraction pattern, just as the earlier observed X-radiation does. The ‘interference fringes’ of the diffraction pattern emerge when the momentum, p, of the electron is related to the *de Broglie wavelength* $\lambda = h/p$, where h is Planck’s constant, and the magnitude of p is such that λ is the order of magnitude of the lattice spacing of the diffracting crystal. [This relation between a (discrete) particle variable – its momentum p – and a (continuous) wave variable - its wavelength λ - was postulated by Louis de Broglie, three years before the experimental discovery.[6]]

The (discrete) particle, electron, was discovered 25 years earlier by J. J. Thomson (the father of G. P. Thomson) is his cathode ray experiments. Yet, the conclusion about the discreteness of the electron from the cathode ray experiment was indirect. This is because one never sees a truly discrete object (in any observation)! What one sees, such as in J. J. Thomson's experiment, is a localized but *slightly smeared* 'spot' on the phosphorescent face of the cathode ray tube. One then extrapolates from this 'spot' to the existence of an actual discrete point where the electron is said to land on the screen. Nevertheless, a close examination of this *smeared spot* would reveal that inside of it, there is indeed a diffraction pattern! Thus, another possible interpretation of the experiments whereby one thinks that one is seeing the *effects of* a discrete particle is that what is actually seen is a 'bunched' continuous wave – that there is no discrete particle in the first place!

The discovery of the wave nature of the electron was a momentous and revolutionary discovery for physics. It signified a possible *paradigm change* in our ontological view of matter, from the *atomistic*, particularistic model that has held since the ancient times, to a continuum, *holistic* model. In the former view, macroscopic matter is viewed as a collection of singular, elementary bits of matter that may or may not interact with each other to affect the physical whole. In contrast, in the continuum, holistic view, there is a single continuous matter field. What is thought of as its individual constituents is in this view a set of manifestations (modes) of this continuum, that is, in principle, the universe! These manifestations may be electrons, or trees or human beings or galaxies. They are all correlated aspects of a single continuum - they are *of* its infinite set of modes, rather than things *in it*.

In the 1920s, when the continuous wave nature of the electron was discovered, the physics community was not willing to accept this paradigm change, from particularity to holism and continuity of the material universe. Instead, mainly under the leadership of N. Bohr, M. Born and W. Heisenberg, (the *Copenhagen school*), they opted to declare a philosophical view of *positivism*. The view was to assert that if an experiment, using macroscopic equipment should be designed to look at micromatter, such as the electron, as a (discrete) particle, as in the cathode ray experiment, this is what the electron would be *then*. But if a different sort of experiment were designed to look at the electron as a (continuous) wave, as in the electron diffraction study, this is what it would be *under those circumstances*. In other words, the type of measurement that is made on it by a macroscopic observer determines the nature of the electron (or any other material elementary particle), even though the continuous wave and discrete particle views logically exclude each other! This *positivistic* epistemological concept claims that all that can be claimed to be meaningful is what can be experimentally verified at the time a measurement is carried out. Thus it is said that both the 'wave' aspect and the 'particle' aspect of the electron are true, though in different types of measurements. This is called "wave-particle dualism". It is the basis of the theory called "quantum mechanics", that was to follow for describing the domain of elementary particles of matter.

Inertial Mass from General Relativity

The *correspondence principle* has been an important heuristic in physics throughout its history. I now wish to use this principle in order to show that the most primitive expression of the laws of inertial mass can be seen in a generalization in general relativity of the quantum mechanical equations in special relativity. We will then extend the quantum mechanical equations in special relativity to derive the field equations for inertia in general relativity.

The equations we start from are the *irreducible* form of quantum mechanics in special relativity – the two-component spinor form (called the Majorana equations). This is irreducible in terms of the underlying symmetry group of special relativity – the *Poincare group*. The latter is a set of only continuous transformations (i.e. without any discrete reflections in space or time) that leave the laws of nature covariant in all inertial frames of reference, *from the perspective of any one of them*. It is the following set of two coupled two-component spinor equations: (units are chosen with $h/2\pi = c = 1$)

$$(\sigma^{\mu}\partial_{\mu} + I)\eta = -m\chi \quad (1a)$$
$$(\sigma^{\mu *}\partial_{\mu} + I^{*})\chi = -m\eta \quad (1b)$$

To restore reflection covariance, one may combine the two spinor field equations (1ab) to yield the single four-component Dirac equation in terms of the bispinor solution, where the top two components are $(\eta + \chi)$ and the bottom two components are $(\eta - \chi)$.

But the more primitive form of the quantum mechanical equations in special relativity, based on the irreducible representations of the underlying *Poincare symmetry group* – a continuous group without reflections - is in terms of the coupled two-component spinor equations (1ab).

In the wave equation (1a) I is the interaction functional that represents the dynamical coupling of all other matter components of the closed system to the given matter field (η,χ), in accordance with the (generalized) Mach principle. $\sigma^\mu \partial_\mu$ is a first order differential operator, $\sigma^\mu = (\sigma^0; \sigma^k)$, where σ^0 is the unit 2-matrix and σ^k (k = 1, 2, 3) are the three Pauli matrices. [The set of four matrices σ^μ correspond with the basis elements of a quaternion.] Thus, the operator $\sigma^\mu \partial_\mu$ is geometrically a scalar, but algebraically it is a quaternion. I^* is the time reversal (or space inversion) of I and $\sigma^{\mu*} = (-\sigma^0; \sigma^k)$ is the time reversal of σ^μ.

The spinor field equations (1ab) are the *irreducible* form of the quantum mechanical equations in special relativity. In the limit as $v/c \rightarrow 0$, where v is the speed of a matter component relative to an observer and *c* is the speed of light, these equations (and the four-component Dirac equation) reduce to the nonrelativistic Schrodinger equation for wave mechanics.

Our goal is to *derive* the inertial mass of matter *m* from a theory of matter in general relativity. This is instead of inserting *m* into the equations, later to have its numerical values adjusted to the data, as it is done in the conventional formulation of quantum mechanics in special relativity. We accomplish this by 1) setting the right-hand sides of equations (1ab) equal to zero and 2) globally extending the left-hand sides of these equations to their covariant expression in a curved spacetime.

Regarding the latter step, we extend the ordinary derivatives of the spinor fields to *covariant derivatives* as follows:

$$\partial_\mu \eta \;\rightarrow\; (\partial_\mu + \Omega_\mu)\eta \equiv \eta_{;\mu} \qquad (2)$$

where Ω_μ is the "spin affine connection" field. It must be added to the ordinary derivative of a two-component spinor in order to make the spinor field (η,χ) integrable in the curved spacetime. Its explicit form is:

$$\Omega_\mu = (1/4)(\partial_\mu q^\rho + \Gamma^\rho{}_{\tau\mu} q^\tau) q_\rho^*$$

where $\Gamma^\rho{}_{\tau\mu}$ is the ordinary affine connection of a curved spacetime.[2] The quaternion field $q^\mu(x)$ is defined fundamentally in terms of the invariant quaternion metric of the spacetime, $ds = q^\mu dx_\mu$ of the (factorized) Riemannian (squared) differential metric invariant, $ds^2 = g^{\mu\nu} dx_\mu dx_\nu$. The quaternion field q^μ is a 16-component variable that is, geometrically, a four-vector, but each of its components is quaternion-valued. It was found to be a solution of a factorized version of Einstein's field equations. It replaces the metric tensor $g^{\mu\nu}$ of Einstein's formalism.[2] The quaternion q_ρ^* is the quaternion conjugate (time-reversal) to q_ρ.

Thus with $m = 0$ and the global extension of the left-hand side of eq. (1a) as indicated above, the matter field equation becomes:

$$q^\mu \eta_{;\mu} \equiv q^\mu(\partial_\mu + \Omega_\mu)\eta + I\eta = 0$$

Transposing terms we then have:

$$(q^\mu \partial_\mu + I)\eta = -q^\mu \Omega_\mu \eta \qquad (3)$$

If the explicit inertial mass is to be derived from first principles in general relativity, then using the correspondence principle, compared in the special relativity limit with $m\chi$ in eq. (1a), it must come from the spin-affine connection term on the right side of eq. (3). Indeed a mathematical analysis showed that there is a *mapping* between the time-reversed spinor variables as follows:[2]

$$q^\mu \Omega_\mu \eta = \lambda[\exp(i\gamma)]\chi \qquad (4)$$

where $\lambda = (1/2)[|\det\Lambda_+| + |\det\Lambda_-|]^{1/2}$ is the modulus of a complex function and $\gamma = \tan^{-1}[|\det\Lambda_-|/|\det\Lambda_+|]^{1/2}$ is its argument, where $\Lambda_\pm = q^\mu\Omega_\mu \pm$ h.c. and 'h.c.' stands for the 'hermitian conjugate' of the term that precedes it and 'det' is the determinant of the function.

Finally, applying the requirement of gauge invariance to the field theory, with the gauge transformations:

first kind: $\eta \rightarrow \eta\exp(-i\gamma/2)$, $\chi \rightarrow \chi\exp(i\gamma/2)$

second kind: $I \rightarrow I + (i/2)q^\mu\partial_\mu\gamma$,

the phase factor in eq. (3), (using eq. (4) on the right-hand side) is automatically transformed away. The field equation (3) – the global extension in general relativity of eq. (1a) - then takes the form:

$$(q^\mu\partial_\mu + I)\eta = -\lambda\chi \quad (5a)$$

Its time-reversed equation (the global extension of (1b) is:

$$(q^{\mu*}\partial_\mu + I^*)\chi = -\lambda\eta \quad (5b)$$

Gauge covariance is a necessary and sufficient condition for the incorporation of the laws of conservation in the field laws, in the asymptotically flat spacetime limit of the theory. Thus the empirical facts about the existence of conservation laws of energy, linear and angular momentum, in the (asymptotically flat) special relativity limit of the theory, dictate that gauge covariance is a necessary symmetry, in addition to the continuous group symmetry in general relativity (the "*Einstein group*") of the field theory.

We see, then, in using the correspondence principle, comparing the generally covariant field equations (5ab) with the asymptotically flat special relativity limit (1ab), that the function λ plays the role of the inertial mass of matter, *m*. Thus we may interpret the generally covariant equations (5ab) as the defining field relations for the inertial mass of matter.

As we asymptotically approach the flat spacetime limit, equations (5ab) approach equations (1ab) and the generally covariant solutions (η, χ) approach the flat spacetime elements of the Hilbert function space $\{\eta_1,\ldots\eta_k, ..; \chi_1,\ldots\chi_k, \ldots\}$, with the condition of square integrability (and normalization) imposed on these spinor variables. In this (Hilbert space) limit of the formalism, the expectation values of the positive-definite field λ is the set of squared eigenvalues (the mass spectrum formula):

$$\lambda_k^{\,2} = |\langle\eta_k| (-q^{\mu *}\Omega_\mu^{\,+})_a (q^\mu \Omega_\mu)_a |\eta_k\rangle|$$

where the subscript 'a' denotes the asymptotic value of the term in parentheses as the flat spacetime limit *is approached*, and the 'dagger' superscript denotes the 'hermitian conjugate' function.

A few points about the inertial mass field λ should be noted. First, in the actual flat spacetime limit, the spin affine connection Ω_μ vanishes so that in this limit $\lambda_k = 0$. The vanishing of the spin affine connection field occurs only for the vacuum – the absence of all matter, *everywhere*. Thus the derivation from general relativity of the vanishing of the inertial mass $\lambda_k = 0$, where there is no other mass to couple to, is in accordance with the statement of the Mach principle.

A second important point is that, as the modulus of a complex function, λ is positive-definite. This implies that any macroscopic quantity of matter, being made up of these 'elementary' units of matter with positive mass, must itself have only positive mass. The implication is that, *in the Newtonian limit of the theory*, the gravitational force has only one polarization. It is either under all conditions repulsive or under all conditions attractive. In view of the locally observed attractive Newtonian gravitational force, it must then under all circumstances be attractive. This conclusion is in agreement with all of the empirical data on Newton's force of gravity. It has never been derived before from first principles, either in Newton's classical theory of gravitation or in the tensor formulation of Einstein's theory of general relativity. This result implies that in the Newtonian limit of the theory there is no anti-gravity, i.e. no gravitational repulsion of one body from another.

The Oscillating Universe Cosmology

In the generally curved spacetime of the theory of general relativity, the role of the gravitational force is not directly related to the mass of matter, as it is in Newton's theory. As we see in the geodesic equation in general relativity (the equation of motion of a test body) the 'force' acting on a body relates to the 'affine connection' of the curved spacetime. The latter is a *non-positive-definite* field. Thus, the general prediction here is that under particular physical circumstances (of sufficiently dense matter and high relative speeds between interacting matter) the 'gravitational force' can be repulsive. Under other physical circumstances (of sufficiently rarefied matter density and low relative speeds of interacting matter) the gravitational force can be attractive.

This result in general relativity, applied to the problem of the universe as a whole, implies an oscillating universe cosmology. At one inflection point, the matter components of the universe begin to repel each other, dominating the attractive components of the general gravitational force, thence leading to the *expansion phase* of the universe, with the matter continuously decreasing its density. Then, when the matter of the universe becomes sufficiently rarefied, and relative speeds between interacting matter is sufficiently low, another inflection point is reached where the attractive component of the gravitational force begins to dominate and initiates the *contraction phase* of the universe. This continues with ever-increasing matter density until the conditions are ripe again for the repulsion of matter to dominate. The universe then reaches the inflection point once again for a turn around from contraction to expansion. The expansion phase starts again, until the next inflection point when the attractive force takes over once more, and so on *ad infinitum*.

The answer to the question: How did the matter of the universe get into the maximum instability stage at the last 'big bang' (the beginning of the present cycle of the oscillating universe) is then: Before the last expansion started, the matter of the universe was contracting toward this physical stage. This view of the oscillating universe denies the idea of a mathematical singularity at the inflection point – at the *beginning* of any particular cycle of the oscillating universe – that is commonly believed by present-day cosmologists who adhere to the 'single big bang' model.

This cosmology also rejects the present-day model wherein there is an absolute time measure – the 'cosmological time' - measuring the time since the last big bang happened. The latter view of absolute time is incompatible with Einstein's theory of relativity, wherein there is no absolute time measure. It is replaced in relativity theory with a totally covariant description of the universe wherein the time measure (as the space measure) is a function of the reference frame from which it is determined. The universe itself cannot be expressed in terms of an absolute reference frame. In the theory of relativity, there are no absolute frames of reference or time measures.

Summary

I have argued that the basis of the theory of general relativity implies that any material system is necessarily a *closed system*. This in turn implies a *holistic model* of matter, whereby there are no separable, individual particles of matter. It is a view that is compatible with a continuum, rather than in terms of a collection of discrete particles of matter.

The first empirical evidence for this continuum view was the discovered wave nature of matter in the experiments on electron diffraction in the 1920s. The "matter waves" (as they were named by their discoverer in theory, Louis de Broglie) may then be viewed as the infinite number of *correlated* manifestations (modes) of a single continuous whole – in principle the universe. This implies that the inertial mass of any local matter is not intrinsic, but rather it is dependent on all of the other matter of the closed universe (the Mach principle). It also follows that *all other* physical properties of matter, as well as inertial mass, such as electric charge, are not intrinsic, but are also measures of coupling within the closed system of matter. This is the "generalized Mach principle".

It was seen that the formal expression of quantum mechanics in special relativity relates, by means of a *correspondence principle*, to a generally covariant field theory of inertia, in general relativity. The formal expressions of quantum mechanics in special relativity, in accordance with the irreducible representations of the *Poincare* *group,* are a set of two coupled two-component spinor equations. Each is a time reversal (or space reflection) of the other. The mass parameter is conventionally inserted in a way that appears as a mapping between the two sorts of (reflected) spinors. Removing the mass term in the latter expression, and globally extending the rest of the equation to a curved spacetime, based on the symmetry of the *Einstein group* of general relativity, leads to the covariant field theory of inertial mass. With the added symmetry of gauge invariance, the field equation is recovered with a *mass field* appearing where the mass parameter was initially inserted.

The asymptotic limit toward the flat spacetime of the latter (nonlinear) field equations in general relativity for inertia is the formal structure of (linear) quantum mechanics – as a linear approximation. This analysis has then led to the derivation of quantum mechanics (the Hilbert space structure) as a linear approximation for a generally covariant field theory of the inertia of matter.

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