On the Source of Anisotropy in Cosmic Radiation from General Relativity

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Summary. - This note points to the qualitative agreement between recent observations of anisotropy in electromagnetic radiation propagation over cosmological distances [1] and the general predictions of the characteristics of the geodesic in general relativity theory, when it is expressed in its most general (irreducible) form [2]. The latter is based on the irreducible representations of the "Einstein group" - a 16-parameter Lie group signifying only continuous, analytic transformations that underlie the general covariance of the laws of nature. The fundamental metrical field - the 16-component quaternion $q^{\mu}(x)$ - is obtained from a factorization of the differential Riemannian invariant ds^2 . This results in a factorization of the 10-component symmetric tensor formalism of Einstein's equations to a 16-component quaternion formalism. The new formalism predicts a geodesic that entails rotational as well as translational motion, for matter as well as radiation.
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A recently reported experiment revealed anisotropy in the electromagnetic radiation that propagates over cosmic distances[1]. This was indicated in systematic rotation of the plane of polarization of the radiation as it propagates through the cosmos.

While these experimental results refute the "cosmological principle", which asserts that the matter distribution of the universe is isotropic and homogeneous, and thus that the geodesics associated with the universe as a whole must be isotropic, the theory of general relativity itself does not require this. All that it does require is conformity with the principle of general covariance.

The algebraic expression of the underlying principle of covariance is that the irreducible representations of the symmetry group of general relativity must be a continuous group, without reflections. The requirement that in the flat spacetime limit one must recover the laws of conservation of energy, momentum and angular momentum further requires that this set of transformations must be analytic, everywhere, in accordance with Noether's theorem. Thus, the basis functions of the irreducible group of general relativity - the solutions of the field equations - must be 'regular' everywhere, that is, without singularities, according to Einstein's requirement of general relativity.

The underlying symmetry group of general relativity is then a 16-parameter Lie group, in its irreducible form. It is the "Einstein group". The 16 essential parameters of this group are the continuous, analytic, spacetime-dependent coefficients, $(\partial x^{\mu\prime}/\partial x^{\nu})(x)$ $(\mu, \nu = 0, 1, 2, 3)$.

The standard tensor form of Einstein's field equations is not only covariant with respect to the "Einstein group", as it should be in general relativity, but it is also covariant with respect to reflections in space and time. Thus, the geodesics associated with this (overly symmetric) field theory does not permit anisotropy, as one would have in an intrinsic rotational motion. But when the reflection symmetry elements are removed from the underlying symmetry group, since they are not required by the theory of general relativity, Einstein's tensor field equations factorize[2]. Instead of the 10-component symmetric second-rank tensor for the metrical field, that is reflection covariant, one arrives at a 16-component metrical field, as the basis functions of the Einstein group, that is not reflection covariant. The latter is a four-vector field, $q^{\mu}(x)$, wherein each of the components of this vector field is quaternion-valued, with four independent components, rather than the real number-valued tensor field. The quaternion metrical field transforms as the direct product of two first-rank spinor fields, $\psi \otimes \psi^*$.

Further, when one solves the variational problem,

(1) $$\delta\int ds \quad = \quad \delta\int q^{\mu}\, dx_{\mu} \; = \; 0$$

the geodesic equation is determined that is quaternion-valued, since the invariant metric of the spacetime, $ds = q^{\mu} dx_{\mu}$, is a quaternion scalar. The geodesic equation is then a set of four independent equations:

$$[d x^{\mu} /ds + \Gamma^{\mu}{}_{\lambda\rho} (dx^{\lambda} /ds)(dx^{\rho}/ds)]_{\alpha\beta} = 0 \tag{2}$$

where (α, β = 1, 2). This quaternion equation of motion then represents four real number-valued field equations for the motion of 'free' matter or radiation in the Riemannian spacetime, rather than the single equation encountered in the standard formalism. This generalization results, in the final analysis, from the reduction of the "reducible representations" of the Einstein group that underlie the symmetry of Einstein's tensor field equations, to the "irreducible representations" of this group, that lead to the factorized quaternion field formalism [3]. This is entirely analogous to the factorization of the scalar Klein-Gordon equation of the pair of conjugated spinor equations (of the Majorana form), when the reflection symmetry elements are removed from the underlying Lorentz group, thereby yielding the Poincare group, which is all that is required to represent the symmetry elements of special relativity. In the latter case, one then has the extra spin degrees of freedom, as one has the extra degrees of freedom in the quaternion field of general relativity, with 16, rather than the10 independent components of the symmetric, second-rank tensor formalism.

The generalized geodesic equation (2) in general relativity then implies, because of the extra degrees of freedom in the metrical field, that a natural motion of a test body, or that of propagating radiation, not only entails translation along a geodesic of the curved spacetime, but it also entails a rotation along this path. It then leads to a natural generalization of *Galileo's principle of inertia,* from the assertion that a 'free body' must move naturally at a constant speed in a straight line (i.e. the geodesic of a Euclidean space), to the assertion that it must move along a curve (a geodesic of a Riemannian spacetime) *with rotation*.

This generalization of the geodesic also implies that electromagnetic radiation, in propagating 'freely' along the curved geodesics of a spacetime expressed with the quaternion irreducible form of the theory of general relativity, must move in a way that its plane of polarization rotates, as it was observed by Nodland and Ralston [1]. It should be emphasized here that the axis referred to in regard to the rotation of the plane of polarization of an electromagnetic propagating wave, is not fixed in any absolute direction. Its orientation is a function of the observer's frame of reference, since this entire formalism is covariant.

Summing up, it may be said at this juncture, at least in qualitative terms, that the discovery of Nodland and Ralston of a systematic rotation of the plane of polarization of electromagnetic radiation, propagating over cosmic distances, is generally predicted by the theory of general relativity, *in its irreducible form.* It follows here because the general form of the geodesics of a curved spacetime is in terms of a quaternion structure. This entails not only translational motion but also rotational motion, as the purely continuous changes that signify the general transformations between reference frames that leave the laws of nature covariant according to the theory of general relativity. It is a generalization from *Galileo's principle of inertia* to Einstein's general relativity, wherein the natural motion is along the most general form of the geodesics of spacetime, that entails rotation as well as translation.

REFERENCES

[1] NODLAND B. and RALSTON J. P., *Phys. Rev. Lett.* **78** (1997) 3043.

[2] SACHS M, *Nuovo Cimento* **47** (1967) 759.

[3] This generalization of the expression of general relativity is developed in detail in my book, SACHS M., *General Relativity and Matter* (Dordrecht, 1982). (GRM).

I have also discussed this quaternion generalization in: SACHS M., *Nature* ***226*** (1970) 138.

It is interesting to note that the discoverer of quaternions, W.R. Hamilton, speculated in the 19th century that the quaternion must play a role as a generalized time parameter in equations of motion. [I have quoted Hamilton's comment on this in GRM, p. 63.] This is indeed what was found in the interpretation of the geodesic equation as an equation of motion, in the quaternion form (2), in this author's analysis in general relativity.