The Legacy of Einstein and Bohr*

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**Dedicated to the 100th anniversary of Einstein's publication of his theory of special relativity.*

A unique occurrence in the History of Physics was the confluence of two simultaneous scientific revolutions in the 20th century – the theory of relativity and the quantum theory. Based on these developments, it is interesting for the future of physics that when examined in terms of their conceptual and mathematical bases, these theories are incompatible. On the other hand, each of these theories requires an incorporation of the other to proceed toward its completion. This is the dilemma we face in these early decades of the 21st century: Which of the two theories should be abandoned and which should be maintained?[1]

Albert Einstein was the leading proponent of the theory of relativity and its philosophy, Niels Bohr was the leading proponent of the quantum theory and its philosophy – as fundamental truths about matter.[2] Because of the incompatibility of the mathematical, philosophical and conceptual bases of these theories, in fundamental terms, as will be discussed in detail later on, they cannot peacefully coexist. But because aspects of each of these theories are necessary to complete the other, the form of the accepted theory must be generalized in such a way that it yields, as a mathematical approximation, the form of the theory whose concepts are abandoned. This is an appeal to the well-known *principle of correspondence*.

Bohr appealed to this principle in showing how the formal expression of quantum mechanics goes smoothly into the structure of classical mechanics as the fundamental unit of action – Planck's constant h – approaches zero, compared with the values of classical action. Similarly, the *principle of correspondence* reveals how Einstein's relativistic mechanics goes smoothly into classical Newtonian mechanics when the speed of light *c* approaches infinity, compared with the relative speed between interacting bodies, *v*. The latter corresponds to the reversal of relativity's requirement that forces propagate at a finite speed (whose maximum value is *c*) with the classical idea of *action at a distance*.

However, the fundamental constants *h* and *c* are not respectively zero and infinity, even though one may approximate them so under particular circumstances. Thus, in principle the theory of relativity and the quantum theory do not *contain* the formal expressions of classical mechanics! *Action at a distance*, a feature of Newtonian mechanics, is not a true concept. It is only that, with the use of the *correspondence principle*, one may use the classical theories as good approximations for a totally different theory of matter, for calculational purposes.

Thus we see that the formal expression of quantum mechanics may be a particular mathematical approximation for a totally different theory, both conceptually and mathematically, such as a theory of matter based on general relativity.

What I wish to do now is to emphasize the major differences between the quantum theory, as understood by Bohr, and the theory of relativity, as understood by Einstein, as conflicting fundamental approaches to our understanding of matter in all domains, from that of elementary particles to cosmology. I will argue that it is indeed the theory of general relativity that will survive, to be explored further in the 21st century, while abandoning the bases of the quantum theory. *This is opposite to the consensus opinion in physics today*! In this regard it is interesting to quote from Maimonides (*The Guide of the Perplexed*, Chicago, 1963*) "For when something has been demonstrated, the correctness of the matter is not increased, and certainty regarding it is not strengthened by the consensus of all men of knowledge with regard to it. Nor could its correctness be diminished and certainty regarding it be weakened even if all people on Earth disagreed with it."*

Still it must be recognized that the formal expression of the quantum theory has been quite successful in its *description* of molecular, atomic and elementary particle physics, at nonrelativistic energies, in the microscopic domain of matter. Thus it may be concluded that general relativity theory must be generalized in such a way that it incorporates the successful predictions of the quantum theory in the microscopic domain of matter, as a mathematical approximation for a true expression in general relativity.

The Wave Nature of Matter

Physics is one of the fundamental sciences - it is the science of inanimate matter. Thus its truths depend on the empirical facts (in addition to mathematical, and conceptual consistency) to support its alleged *explanations* of the nature of matter. (*To say that a particular theoretical scheme does not have a any empirical content, is to deny that this is indeed a theory of physics, that it has anything to do with the real world – no matter how beautiful this scheme may seem to its inventors*!) The single most important empirical fact that led to the quantum theory was the observation that a material particle – an electron – has a wave nature. This is based on the observations in the 1920s of G. P. Thomson in the UK and Davisson and Germer in the US, that electrons diffract from a crystal lattice, the same way that radiation does (as observed in the earlier X-ray diffraction studies). [4]

At the turn of the 20th century, J.J. Thomson (the father of G. P. Thomson) discovered the smallest elementary particle – the electron. He saw it as a discrete particle in his cathode ray experiments. In this experiment, he measured the location of a spot on the face of a cathode ray tube, directed with electric and magnetic fields of force.[5] But 25 years later, it was seen in experimentation that a beam of electrons (with a 'wavelength' that is the order of the de Broglie wavelength $\lambda = h/p$, where p is the electron's momentum) will diffract from the crystal lattice if the lattice spacing is the order of magnitude of λ. That is to say, the electrons were seen to scatter from the crystal lattice with a diffraction pattern – with interference maxima and minima. That is to say, there are locations on a target screen where the scattered electrons land, where the electrons bunch together and other places where no electrons land. (These are the respective positions of constructive and destructive interference). If the electrons were indeed discrete particles of matter, one would expect them to land on the screen so as to reveal the geometrical pattern of the crystal lattice. They would not reveal a diffraction pattern as though they were waves.

If we look back at J. J. Thomson's experiment, that seemed to reveal the discreteness of the electrons, we would see a 'spot' on the phosphorescent face of the cathode ray tube that while highly localized is indeed *not* a discrete spot! If one should look into this spot with sufficiently high resolution, a diffraction pattern would be seen inside of it. It could then be argued that indeed the electron is a continuous wave, under all conditions, even under those empirical conditions where it seems to be a discrete particle, though in reality it is a 'bunched' wave! The reason given by the Copenhagen school for the 'unavoidable' diffraction pattern in the diffused spot that is supposed to be a discrete electron is the *Heisenberg uncertainty principle.*[6] The latter claims that the measurement of even a single electron must interfere with the measuring apparatus that 'looks at it', so as to generate more waves that then interfere with the original electron wave. According to this principle, then, it is impossible to locate the electron's precise position if there can be precise knowledge of its momentum p. Thus, the claim is that we should expect to see the diffraction pattern in the 'smeared, (though bunched) spot' that is supposed to be the discrete electron.

Leading atomic physicists at the time when the electron diffraction experiments were carried out, in the 1920s, were Bohr and Heisenberg. They insisted on the particle nature of the electron (and all other elementary particles) in spite of its observed wave nature. Believing that under the conditions of J. J. Thomson's experiment (and other experiments that they believed revealed the discreteness of the electron), as well as knowing about the revealed wave nature of the electron, they enunciated the concept of 'wave-particle dualism' to underlie the true nature of matter. Einstein had already introduced this concept to understand the seemingly wave and particle nature of the quantum of electromagnetic radiation – the photon - (a concept that Einstein abandoned later on!).

The concept of 'wave-particle dualism' asserts that if one should perform an experiment designed to see the electron as a discrete particle, it would be a discrete particle at that time of measurement. But if at a different time one should perform an experiment designed to see the electron as a wave, it would be a wave at that time. In other words, the 'electron' is whatever the experimental set-up sees it to be, particle or wave, even though the fundamental nature of particle and wave are logically dichotomous. This idea is consistent with the epistemological view of *logical positivism*. It is a philosophical view that was predominant in thinking in the early decades of the 20th century.[7]

The view of *logical positivism* is based on the ‘principle of verifiability’. This is the notion that only those scientific concepts that are verifiable in terms of direct human observations or measurements are meaningful truths in nature. *A well known refutation of this principle by Bertrand Russell is the following: The principle of verifiability is itself not verifiable by direct observations or measurement. Thus, if it is true it must be false. Therefore the principle is false.*[8]

The next stage in the development of quantum theory was to properly interpret the wave that is the electron (or any other of the elementary particles), and to find the law that determines the explicit wave form of the particle. Erwin Schrödinger found this law in the form of the Schrödinger wave equation. This formulation led to ‘nonrelativistic wave mechanics’ – for particles that were moving at speeds small compared with the speed of light. It was then recognized by Max Born that this formulation could be put into the form of a probability calculus. Thus, it was Born who interpreted the wave function as a wave of probability. The Copenhagen school, led by Bohr, then said that this probability relates to the *measurement* of the physical qualities of the particle of matter.

On the interpretation of the ‘wave function’ ψ, for the electron, Schrödinger did not accept the Copenhagen view. He believed the wave nature of the electron was to complete the form of Maxwell’s field theory of electromagnetism. In the Maxwell field theory, the sources of the electric and magnetic field solutions are the (real number) electrical charge and current densities. As real numbers, they cannot represent a propagating wave - that must necessarily have a complex function representation. What Schrödinger proposed was that the real number charge density (and the corresponding current density) is the product of the matter wave field ψ and its complex conjugate, to yield the real number density $\varrho = e\psi^*\psi$, (and a corresponding term for current density) where e is the charge of the electron. In the electron scattering experiments, in the microscopic domain, this unfolds to the matter wave ψ itself.

Summing up, both Born (and Bohr), on the one hand, and Schrödinger (and Einstein), on the other, agreed that the electron wave function comes from a factorization of the electric charge density $\varrho = e\psi^*\psi(x)$. But according to Bohr’s school, the wave function ψ is fundamentally related to the measurement process that entails *a single particle* of matter and it is rooted in a probability calculus. This represents the probability of locating the electron at the position x. On the other hand, in Schrödinger's view, ψ is a matter wave (not unlike a water wave), that represents a mode of a matter continuum. It is analogous to one of the normal modes of vibration of a set of blocks connected with springs. It does not represent any single one of these blocks. It was Louis de Broglie who originally postulated the "matter wave", a few years before the experimental discovery of the wave nature of the electron. [9]

On the topic of the complex number nature of the wave function of the electron, it is interesting to note that Schrödinger criticized Einstein's hope that his theory of general relativity would eventually yield the formal structure of the quantum theory, as one of its manifestations. His criticism was based on the fact that the field variables of general relativity are real number valued - the solutions of his equations are the field components of the metric tensor, $g^{\mu\nu}$ - while the wave function of quantum mechanics is necessarily complex-number valued, to describe a propagating wave. This is in addition to the fact that, as a probability calculus, quantum mechanics is necessarily a mathematically *linear* theory. A fundamental feature of the quantum theory, as a probability calculus, is the *principle of linear superposition* – implying that any linear sum of the solutions of the quantum equations is another possible solution. On the other hand, the formal expression of Einstein's general relativity is necessarily a mathematically *nonlinear* formalism, thereby automatically violating the linear superposition principle of the quantum theory and its interpretation in terms of probabilities.

But we do see an answer to Schrödinger's criticism in the full exploitation of Einstein's theory of general relativity. Einstein stressed in his writing that in fully exploiting the theory of general relativity, one must study the algebraic logic as well as the geometric logic in the full expression of the theory. The algebraic logic entails the underlying symmetry group of the theory. In regard to the symmetry group that underlies the principle of covariance of the theory of relativity, its irreducible representations obey the algebra of quaternions; their basis functions, in turn, are spinor field variables. The quaternion and spinor field variables, in turn, are made up of components that are complex number valued. Thus, the basic field variables of matter, according to the theory of relativity, in any domain, from that of particle physics to cosmology, must obey the algebra of quaternions and spinors, with their complex number components. These variables are not, generally, the real number valued components of the metric tensor $g^{\mu\nu}$ of Einstein's original form of his field equations.[10] They are functions whose components are complex number valued. This result then allows the incorporation of the formalism of complex number valued quantum mechanics to be incorporated in the expression of general relativity.

Why does this factorization to the quaternion-spinor formalism follow from the 'irreducible form' of the theory of general relativity? It is because Einstein's equations are more symmetric than they need be, according to its underlying group required by the principle of covariance. These equations are covariant with respect to the continuous transformations of space and time coordinates, as required. But they are also covariant with respect to the discrete reflections of the space and time coordinates, *which is not required by the theory*. Thus, by removing the discrete reflections from the underlying symmetry group of general relativity theory, Einstein's tensor field equations factorize to a quaternion form, thereby yielding the complex number variables required by the quantum theory. Thus the degeneracy of the former form of the equations – 'Einstein's field equations' - is lifted and the 10 independent equations unfold to a set of 16 independent equations. The latter are then no longer covariant with respect to space and time reflections. Thus we see that the most general form of the theory of general relativity is amenable to the incorporation of the formal structure of the quantum theory, as one of its general features. Later on, I will demonstrate how the formal expression of quantum mechanics comes out as a linear approximation for a generally covariant (nonlinear) field theory of the inertia of matter in general relativity.

In my view, the primary assertion of the Copenhagen School is Bohr's *principle of complementarity.*[11] This is a generalization of the wave-particle dualism concept, as we have discussed earlier. It is a notion compatible with the epistemological view of *logical positivism*, that logically opposing views are equally admissible as true, so long as they are not revealed (by measurement or direct observation) at the same time, under the same conditions of experimentation. This *positivistic* view was stated explicitly by a member of the Copenhagen School, one of the founders of quantum mechanics, Werner Heisenberg. In the opening section of one of his initial papers on quantum mechanics he said: "*The present paper seeks to establish a basis of theoretical quantum mechanics founded exclusively upon relationships between quantities which in principle are observable".* [12]
In contrast, the theory of relativity is based on the epistemological notion of *realism* – the idea that the ways of the world are fully objective, not dependent on whether or how an observer measures its qualities.

Complementarity is then a philosophical view of *pluralism.* That is to say, the fundamental truths that we seek, according to the Copenhagen school, are multi-valued. This means that mutually incompatible assertions can be simultaneously accepted as true, as long as their determinations occur under different sets of physical circumstances. The macro-observation then plays a fundamental role in the *definition* of the elements of matter and radiation. They are not, in this view, 'things in themselves'! Thus, we may say that an entity, such as an electron, can be truly a discrete particle, under the experimental conditions that see it so, and at the same time an electron can be truly a continuous wave under different types of experimental conditions that see it this way. I have argued that there is no direct empirical evidence to support the model of the electron as a truly discrete particle. But there is empirical support for the wave nature of the electron.

In contrast with this view, the theory of relativity, based on the *principle of relativity* (also known as the *principle of covariance*) is based on a philosophical view of *monism.* This principle implies that there is a single explanatory base – a single underlying order. This order - an order of the universe - is expressed in terms of the laws of nature, laws whose forms are independent of any frame of reference in which they are described, from the view of any observer. That is to say, it is the assertion of the theory of relativity that the laws of nature must be fully objective. *In contrast*, *the quantum theory entails an irreducible subjectivity, because it does indeed entail an absolute reference frame – that of the measuring apparatus.* That is to say, in the quantum theory there is an 'observer' (the 'absolute' measuring apparatus) and an 'observed' (the microscopic matter that the equations of quantum mechanics refer to). There is no causal connection between the observer and the observed according to this theory; we interpret the outcomes of measurements in terms of probabilities for the values of the physical qualities of matter. Thus, the variables of the observer and the observed are not the same – those of the observer (e.g. human observers and their measuring apparatuses) are classical (macro-) variables, while those of the observed (the microscopic matter) are the quantum mechanical (micro-) variables. In the theory of relativity, on the other hand, there is no difference between the observer and the observed; in principle they both entail the same dynamics and types of variables. Thus the theory must be symmetric to the interchange of the observer and the observed. This is a holistic theory of matter that automatically rejects Bohr's pluralistic principle of complementarity. In relativity theory, there is a single explanatory level for the workings of the universe, in regard to any of its manifestations – from the smallest to the largest, from the domain of elementary particle physics to that of cosmology.

The Philosophy of the Theory of Relativity

Einstein's theory of relativity is based on one central assertion – the principle of relativity (also called 'the principle of covariance'). It is none other than the assertion that the laws of nature must be fully objective. This means that the expression of any law of nature must be independent of the frame of reference in which it is expressed by any observer – that is to say, the expressions of any law of nature in all possible reference frames, from this observer's perspective, must be in *one-to-one correspondence.*

It is in the ‘expression’ of the law of nature where the space and time come in, as the ‘words’ of a language. It is the spacetime language that is *relative* to the frame of reference in which the laws of nature are expressed. This is a language that is structured in order to facilitate an expression of the laws of nature. The space and time parameters are then the independent variables of a grid that is there in order to *map* the dependent variables – that is, the solutions of the laws of nature. In a field theory, such as Maxwell’s theory of electromagnetism, the dependent variables would be the electric and magnetic field variables $\mathbf{E}(x, y, z, t)$ and $\mathbf{H}(x, y, z, t)$. In a particle theory, such as Newton’s, these would be the position variables of a thing of matter, $\mathbf{r}(t)$. This is analogous to the relativity of verbal languages (to cultural, geographic, etc. reference frames) that express invariant meanings in their sentences.

In the theory of general relativity, the structure of the space and time language grid is determined by the matter content of the universe. Because the matter content of the universe (its field representation) is generally continuously variable, the ‘metric’ of the spacetime must be correspondingly continuously variable. In Einstein’s version of general relativity, the (squared) differential metric then has the Riemannian form:

$$ds^2 = \Sigma_{\mu\nu} g^{\mu\nu}(x) dx_\mu dx_\nu$$

where the sum is taken over $\mu, \nu = 0, 1, 2, 3$, (0 is the time coordinate and 1,2,3 are the spatial coordinates) and x refers to the four space and time coordinates – the grid of independent variables in which the metric tensor $g^{\mu\nu}$ is mapped. It is then asserted that the covariance of the laws of nature is based on the same spacetime transformations that keep ds^2 invariant with respect to changes of reference frames, i.e., for $x \rightarrow x'$, $ds^2 = ds'^2$. In the limit as matter approaches a perfect vacuum, *everywhere*, ds^2 approaches the metric of special relativity theory. ds^2 generally entails a curved spacetime, while that of special relativity entails a flat spacetime. The curved spacetime describes the existence of matter in the universe, *anywhere*. It is only in the (ideal) limit of a perfect vacuum *everywhere*, that $g^{00} \rightarrow 1$, $g^{kk} \rightarrow -1$, $g^{\mu\neq\nu} \rightarrow 0$ (where $k = 1, 2, 3$ are the spatial coordinates), so that in the limit of a perfect vacuum, we approach the special relativity *flatspace*(squared) metric $ds^2 = dx^{0\,2} - dr^2$. Thus, in principle, the theory of special relativity refers only to the ideal limit of a perfect vacuum, *everywhere*.[13]

Einstein showed that the metric tensor solutions $g^{\mu\nu}(x)$ of the nonlinear differential equation, on the left hand side of the equation, depends on the *source terms* $T^{\mu\nu}(x)$ on the right side of the equation, representing the energy-momentum of the matter of the closed system (in principle the universe) that gives rise to metrical features of the spacetime. These are 10 independent nonlinear differential equations of second order. They are the 'Einstein field equations'. Einstein then showed that the curvature of spacetime determined by this metric tensor gives rise to the prediction of the force of gravity. [14]

Because the principle of relativity requires a duplication of the forms of the laws of nature under the continuous transformations of the space and time coordinates $x \rightarrow x' = x + \delta x$, the solutions of these equations must be continuous functions of x, *everywhere.* In addition, it was shown by E. Noether that the transformations that preserve the forms of the laws of nature (i.e. to preserve covariance) must not only be continuous, but also analytic – that is, the derivatives of the four coordinates $x^{\mu'}$ with respect to the continuously connected four coordinates of a different reference frame x^{ν} must exist to all orders. This ensures the existence of the laws of conservation of energy, momentum and angular momentum, in the flat spacetime limit of the theory. (*Noether's theorem*).[15] Thus, the group of transformations that underlies the covariance of the laws of nature – the Einstein group - is a *Lie group*. The number of essential parameters of this group is the number of derivatives $\partial x^{\mu'}/\partial x^{\nu}$ – this is the number $4 \times 4 = 16$. The number of essential parameters of the Lie group, 16, then determines the number of independent relations (equations) at each spacetime point. But Einstein's equations are only 10 in number. Why is this? It is because Einstein's equations are more symmetric than they need to be. For they are not only covariant with respect to the continuous transformations, as required – the Lie group of general relativity, i.e. the 'Einstein group' - they are also covariant with respect to the discrete reflections of the space and time coordinates, which is not a requirement of general relativity. When the reflections are removed from the underlying covariance group of Einstein's tensor equations, they factorize to quaternion field equations. The new 16-component field variable $q^{\mu}(x)$, is a four-vector, in which each of its four component is quaternion valued. This 16-component field, covariant under all continuous transformations but not the discrete reflections, then replaces the 10-component metric tensor $g^{\mu\nu}$ as the fundamental metrical field.

It then turned out that of the 16 independent real number variables, 10 relate to the gravitational force manifestation (as Einstein showed in his original tensor form of general relativity) and 6 relate to the Maxwell field equations, yielding the field variables of electromagnetism (3 components of the electric field and 3 components of the magnetic field). Thus we have a *unified field theory* that entails gravity and electromagnetism in a single 16-component quaternion metrical field $q^{\mu}(x)$.[16]

Quantum Mechanics from General Relativity

Finally, let me indicate in non-mathematical terms how the formal structure of quantum mechanics emerges from General Relativity. *It is worked out in detail in* [3].

The idea is to first identify a generally covariant theory of the inertia of matter from general relativity theory. This starts with the most primitive form in physics where the inertial mass of matter appears, that is in the microscopic domain. (Once the mass of micro-matter is derived from such a theory, the masses of macro-quantities of matter may be built up from this). With adherence to the underlying symmetry group of relativity theory, that is the group of continuous transformations without reflections; this, in special relativity, is the Majorana form of the Dirac equation for relativistic wave mechanics. It takes the form of two coupled spinor equations, in terms of the reflected two-component spinor variables η and χ. In these equations the mass appears as a factor multiplying one of the spinors on the right side of the equation, $m\chi$, while the left side of the equation is a quaternion operator acting on the reflected spinor, η. The second equation is a reflection (in space or time) of this equation.

The procedure then, to derive the mass associated with this spinor field, is to 1) set the right side of this equation, $m\chi$, equal to zero and 2) re-express the left hand side of the equation in a curved spacetime. When this is done, it is found that, indeed one arrives at the exact form of wave mechanics – in the Majorana form of the Dirac equation – in a curved spacetime. The flat spacetime limit of this equation is then the Majorana/Dirac form of relativistic quantum mechanics in special relativity, and the nonrelativistic limit of the latter equation is the Schrödinger form of wave mechanics.[17]

The term in the general form of this equation, in general relativity, that plays the role of the mass of matter, is a positive-definite function of the quaternion variables and the 'spin-affine connection' of the spacetime, derived in general relativity. The 'spin affine connection' is a necessary term that allows integrability of the spinor solutions in the curved spacetime.

The 'spin affine connection' field, and therefore the inertial mass of matter, vanishes in the limit of the flat spacetime. Thus, in accordance with the *Mach principle*, the mass of elementary matter depends on its coupling with all of the other matter of a closed universe. In the limit of the approximation of a flat spacetime, in special relativity, *where the curvature of spacetime is nevertheless there in the background*, one may insert a non-zero, averaged mass field m in the matter field equations.

Thus we see that by fully exploiting the symmetry group of the theory of general relativity, one arrives at a unified field theory that incorporates gravity and electromagnetism, in a generally covariant field theory, and automatically incorporates a nonlinear field theory of the inertia of matter. The linear (flat spacetime) approximation for this theory of inertia is the full (Hilbert space) structure of quantum mechanics. This was one of Einstein's anticipations - that general relativity, properly generalized, would yield the form of linear quantum mechanics as an approximation for a nonlinear field theory of matter. We have seen that the key to unification was the incorporation of the inertia of matter in the full field formalism of general relativity theory.

Concluding Remarks

Summing up, in my view the legacy of Einstein and Bohr in 21st Century physics is a unification of the field theories of gravitation and electromagnetism in terms of a 16-component, nonsingular quaternion/spinor field formalism that also incorporates the full expression of quantum mechanics as a linear approximation for a generally covariant field theory of the inertia of matter. The conceptual bases of quantum mechanics – atomism, indeterminism, linearity and logical positivism are replaced by the conceptual bases of the theory of general relativity – continuity and holism, determinism, nonlinearity and realism. It is my belief that our future comprehension of physics in the 21st Century will proceed from these bases of the theory of general relativity.

References

[1] A detailed account of the incompatibilities of the concepts of the relativity and quantum theories is given in: M. Sachs: *Einstein Versus Bohr* (Open Court, 1988), Chapter 10.

[2] The major argument between Einstein and Bohr on the philosophical bases of the quantum and relativity theories is expounded in: P. A. Schilpp, editor: *Albert Einstein: Philosopher-Scientist* (open Court, 1949).

[3] I have developed a theory based on general relativity that incorporates a theory of the inertia of matter. The asymptotic, low energy limit of the latter theory is the formal expression of quantum mechanics. This is shown in my three books: M. Sachs, *General Relativity and Matter* (Reidel, 1982); M. Sachs, *Quantum Mechanics from General Relativity* (Reidel, 1986) and M. Sachs, *Quantum Mechanics and Gravity* (Springer, 2004).

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[13] M. Sachs, *ibid.*

[14] A. Einstein, *The Meaning of Relativity: Relativity Theory of the Non-Symmetric Form* (Princeton, 1956), 5th edition.

[15] For a mathematical explication of Noether's theorem, see: C. Lanczos, *The Variational Principles of Mechanics* (Toronto, 1966), 3rd edition, Appendix II.

[16] M. Sachs, *Quantum Mechanics and Gravity* (Springer, 2004)

[17] M. Sachs, *Quantum Mechanics from General Relativity* (Reidel, 1986), Chapter 4.